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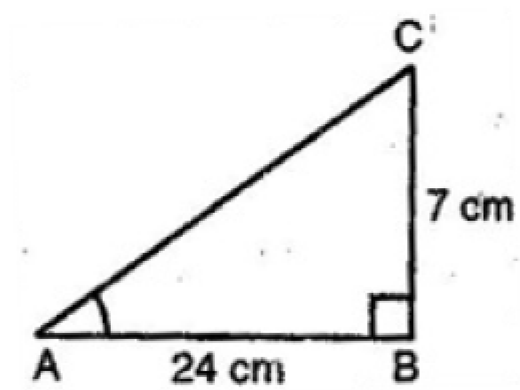
# NCERT Class 10 Maths: Chapter 8 - Introduction to Trigonometry Solutions

**Ex 8.1 Question 1.**

In $\triangle \mathrm{ABC}$, right angled at $\mathrm{B}, \mathrm{AB}=24 \mathrm{~cm}, \mathrm{BC}=7 \mathrm{~cm}$. Determine:
(i) $\sin A \cos A$
(ii) $\sin C \cos C$

**Answer.**

Let us draw a right angled triangle $\mathrm{ABC}$, right angled at $\mathrm{B}$.
Using Pythagoras theorem,

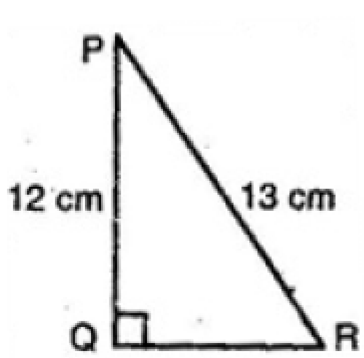



Let $\mathrm{AC}=24\mathrm{k}$ and $\mathrm{BC}=7\mathrm{k}$

Using Pythagoras theorem,
$AC^2=AB^2+BC^2$
$=(24)^2+(7)^2=576+49=625$
$\Rightarrow \mathrm{AC}=25 \mathrm{~cm}$
(i) $\sin \mathrm{A}=\frac{\mathrm{P}}{\mathrm{H}}=\frac{\mathrm{BC}}{\mathrm{AC}}=\frac{7}{25}, \cos \mathrm{A}=\frac{\mathrm{B}}{\mathrm{H}}=\frac{\mathrm{AB}}{\mathrm{AC}}=\frac{24}{25}$
(ii) $\sin \mathrm{C}=\frac{\mathrm{P}}{\mathrm{H}}=\frac{\mathrm{AB}}{\mathrm{AC}}=\frac{24}{25}, \cos \mathrm{C}=\frac{\mathrm{B}}{\mathrm{H}}=\frac{\mathrm{BC}}{\mathrm{AC}}=\frac{7}{25}$

**Ex 8.1 Question 2.**

In adjoining figure, find $\tan P-\cot R$ :



**Answer.**

In triangle PQR, Using Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (13)^2 = (12)^2 + QR^2$

$\Rightarrow QR^2 = 169 - 144 = 25$

$\Rightarrow QR = 5 \text{ cm}$

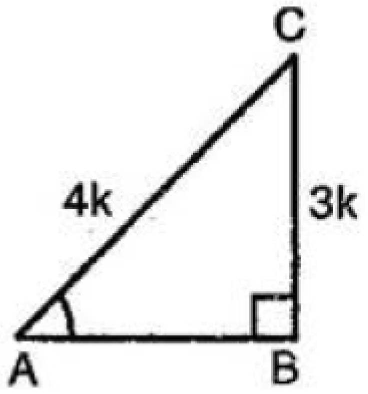
$\therefore \tan P - \cot R = \frac{P}{B} - \frac{B}{P} = \frac{QR}{PQ} - \frac{QR}{PQ} = \frac{5}{12} - \frac{5}{12} = 0$

**Ex 8.1 Question 3.**

If $\sin A = \frac{3}{4}$, calculate $\cos A$ and $\tan A$.

**Answer.**

Given: A triangle $\mathrm{ABC}$ in which $\angle \mathrm{B} = 90^\circ$




Let $\mathrm{BC} = 3k$ and $\mathrm{AC} = 4k$

Then, Using Pythagoras theorem,

$\mathrm{AB} = \sqrt{(\mathrm{AC})^2 - (\mathrm{BC})^2} = \sqrt{(4k)^2 - (3k)^2}$

$= \sqrt{16k^2 - 9k^2} = k\sqrt{7}$

$\therefore \cos \mathrm{A} = \frac{\mathrm{B}}{\mathrm{H}} = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

$\tan \mathrm{A} = \frac{\mathrm{P}}{\mathrm{B}} = \frac{\mathrm{BC}}{\mathrm{AB}} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

**Ex 8.1 Question 4.**

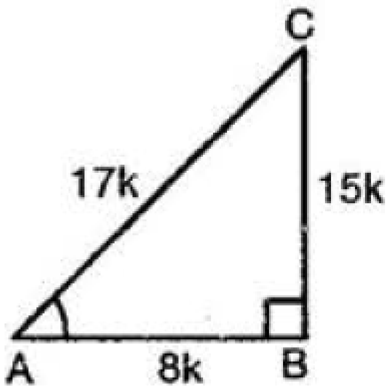
Given $15 \cot A = 8$, find $\sin A$ and $\sec A$

**Answer.**

Given: A triangle $\mathrm{ABC}$ in which $\angle \mathrm{B} = 90^\circ$

$15 \cot A = 8$

$\Rightarrow \cot A = \frac{8}{15}$




Let $\mathrm{AB} = 8k$ and $\mathrm{BC} = 15k$

Then using Pythagoras theorem,

$\mathrm{AC} = \sqrt{(\mathrm{AB})^2 + (\mathrm{BC})^2}$

$= \sqrt{(8k)^2 + (15k)^2}$

$= \sqrt{64k^2 + 225k^2}$

$= \sqrt{289k^2} = 17k$

$\therefore \sin \mathrm{A} = \frac{\mathrm{P}}{\mathrm{H}} = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{15k}{17k} = \frac{15}{17}$

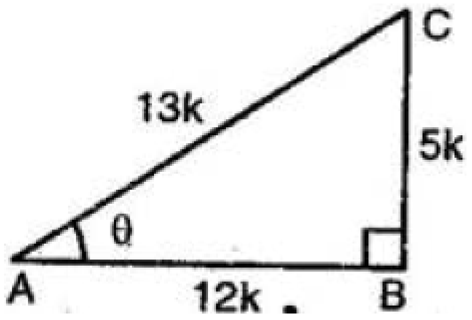
$\sec \mathrm{A} = \frac{\mathrm{H}}{\mathrm{B}} = \frac{\mathrm{AC}}{\mathrm{AB}} = \frac{17k}{8k} = \frac{17}{8}$

**Ex 8.1 Question 5.**

Given $\sec \theta = \frac{13}{12}$, calculate all other trigonometric ratios.

**Answer.**

Consider a triangle $\mathrm{ABC}$ in which $\angle \mathrm{A} = \theta$ and $\angle \mathrm{B} = 90^\circ$




Let $\mathrm{AB}=12k$ and $\mathrm{BC}=5k$

Then, using Pythagoras theorem,

$\mathrm{BC}=\sqrt{(\mathrm{AC})^2-(\mathrm{AB})^2}$

$=\sqrt{(13k)^2-(12k)^2}$

$=\sqrt{169k^2-144k^2}$

$=\sqrt{25k^2}=5k$

$\therefore \sin\theta=\frac{\mathrm{P}}{\mathrm{H}}=\frac{\mathrm{BC}}{\mathrm{AC}}=\frac{5k}{13k}=\frac{5}{13}$

$\cos\theta=\frac{\mathrm{B}}{\mathrm{H}}=\frac{\mathrm{AB}}{\mathrm{AC}}=\frac{12k}{13k}=\frac{12}{13}$

$\tan\theta=\frac{\mathrm{P}}{\mathrm{B}}=\frac{\mathrm{BC}}{\mathrm{AB}}=\frac{5k}{12k}=\frac{5}{12}$

$\cot\theta=\frac{\mathrm{B}}{\mathrm{P}}=\frac{\mathrm{AB}}{\mathrm{BC}}=\frac{12k}{5k}=\frac{12}{5}$

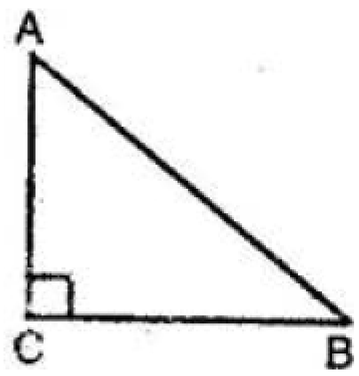
$\cos ec\theta=\frac{\mathrm{H}}{\mathrm{P}}=\frac{\mathrm{AC}}{\mathrm{BC}}=\frac{13k}{5k}=\frac{13}{5}$

**Ex 8.1 Question 6.**

If $\angle$ And $\angle\mathbf{B}$ are acute angles such that $\cos A=\cos B$, then show that $\angle\mathbf{A}=\angle\mathbf{B}$.

**Answer.**

In right triangle $\mathrm{ABC}$,




$\cos A=\frac{\mathrm{AC}}{\mathrm{AB}}$ and $\cos B=\frac{\mathrm{BC}}{\mathrm{AB}}$

But $\cos A=\cos B$ [Given]

$\Rightarrow \frac{\mathrm{AC}}{\mathrm{AB}}=\frac{\mathrm{BC}}{\mathrm{AB}}$

$\Rightarrow \mathrm{AC}=\mathrm{BC}$

$\Rightarrow \angle\mathrm{A}=\angle\mathrm{B}$

[Angles opposite to equal sides are equal]

**Ex 8.1 Question 7.**

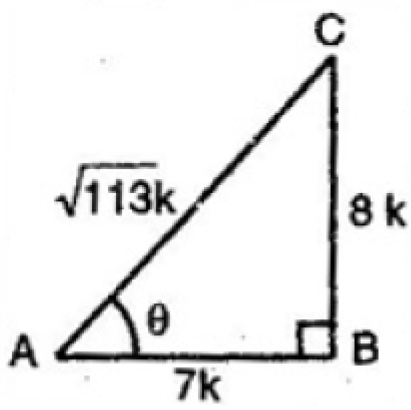
If $\cot\theta=\frac{7}{8}$, evaluate:

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(ii) $\cot^2\theta$

**Answer.**

Consider a triangle $\mathrm{ABC}$ in which $\angle\mathrm{A}=\theta$ and $\angle\mathrm{B}=90^\circ$



Let $\mathrm{AB} = 7k$ and $\mathrm{BC} = 8k$
Then, using Pythagoras theorem,

$$\mathrm{AC} = \sqrt{(\mathrm{BC})^2 + (\mathrm{AB})^2}$$
$$= \sqrt{(8k)^2 + (7k)^2}$$
$$= \sqrt{64k^2 + 49k^2}$$
$$= \sqrt{113k^2} = \sqrt{113}k$$
$$\therefore \sin\theta = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$
$$\cos\theta = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = \frac{113-64}{113-49} = \frac{49}{64}$

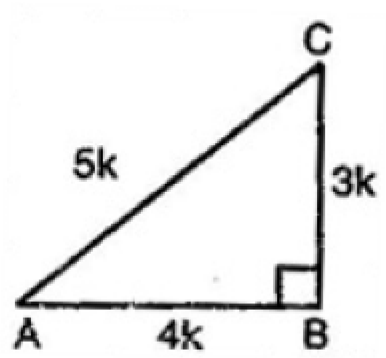
(ii) $\cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{49/113}{64/113} = \frac{49}{64}$

**Ex 8.1 Question 8.**

If $3\cot A = 4$, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

**Answer.**

Consider a triangle $\mathrm{ABC}$ in which $\angle\mathrm{B} = 90^\circ$.




And $3\cot A = 4$
$\Rightarrow \cot A = \frac{4}{3}$

Let $\mathrm{AB} = 4k$ and $\mathrm{BC} = 3k$.
Then, using Pythagoras theorem,

$$\mathrm{AC} = \sqrt{(\mathrm{BC})^2 + (\mathrm{AB})^2}$$
$$= \sqrt{(3k)^2 + (4k)^2}$$
$$= \sqrt{16k^2 + 9k^2}$$
$$= \sqrt{25k^2} = 5k$$
$$\therefore \sin A = \frac{\mathrm{BC}}{\mathrm{AC}} = \frac{3k}{5k} = \frac{3}{5}$$

$\cos A = \frac{\mathrm{AB}}{\mathrm{AC}} = \frac{4k}{5k} = \frac{4}{5}$

And $\tan A = \frac{\mathrm{BC}}{\mathrm{AB}} = \frac{3k}{4k} = \frac{3}{4}$

Now, L.H.S. $\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

$= \frac{16-9}{16+9} = \frac{7}{25}$

R.H.S. $\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

∵ L.H.S. = R.H.S.
$\therefore \frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

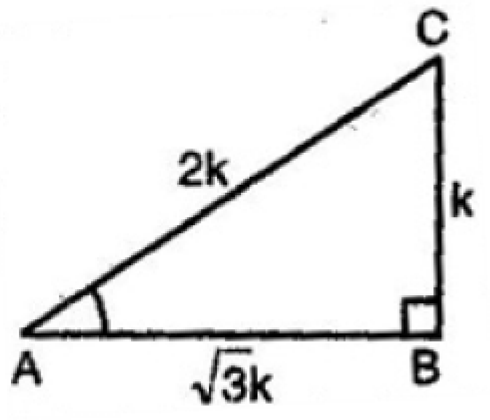
**Ex 8.1 Question 9.**

In $\triangle\mathrm{ABC}$ right angles at $\mathrm{B}$, if $\tan A = \frac{1}{\sqrt{3}}$, find value of:

(i) $\sin A\cos C + \cos A\sin C$
(ii) $\cos A\cos C - \sin A\sin C$

**Answer.**

Consider a triangle $\mathrm{ABC}$ in which $\angle\mathrm{B} = 90^\circ$.
Let $\mathrm{BC} = k$ and $\mathrm{AB} = \sqrt{3}k$




Then, using Pythagoras theorem,

$AC = \sqrt{(BC)^2 + (AB)^2}$

$= \sqrt{(k)^2 + (\sqrt{3}k)^2}$

$= \sqrt{k^2 + 3k^2} = \sqrt{4k^2} = 2k$

$\therefore \sin A = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

For $\angle C$, Base $= BC$, Perpendicular $= AB$ and Hypotenuse $= AC$

$\therefore \sin C = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$

$\cos C = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$

(i) $\sin A \cos C + \cos A \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1$

(ii) $\cos A \cos C - \sin A \sin C = \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$

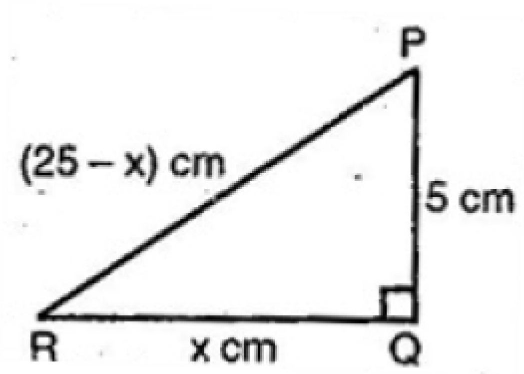
$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$

**Ex 8.1 Question 10.**

In $\triangle PQR$, right angled at $Q, PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P, \cos P$ and $\tan P$.

**Answer.**

In $\triangle PQR$, right angled at Q.




$PR + QR = 25$ cm and $PQ = 5$ cm

Let $QR = x$ cm, then $PR = (25 - x)$cm

Using Pythagoras theorem,

$RP^2 = RQ^2 + QP^2$

$\Rightarrow (25 - x)^2 = (x)^2 + (5)^2$

$\Rightarrow 625 - 50x + x^2 = x^2 + 25$

$\Rightarrow -50x = -600$

$\Rightarrow x = 12$

$\therefore RQ = 12$ cm and $RP = 25 - 12 = 13$ cm

$\therefore \sin P = \frac{RQ}{RP} = \frac{12}{13}$

$\cos P = \frac{PQ}{RP} = \frac{5}{13}$

And $\tan P = \frac{RQ}{PQ} = \frac{12}{5}$

**Ex 8.1 Question 11.**

State whether the following are true or false. Justify your answer.

(i) The value of $\tan A$ is always less than 1 .

(ii) $\sec A = \frac{12}{5}$ for some value of angle A.

(iii) $\cos A$ is the abbreviation used for the cosecant of angle $\mathrm{A}$.
(iv) $\cot A$ is the product of $\cot$ and $\mathrm{A}$.
(v) $\sin \theta = \frac{4}{3}$ for some angle $\theta$.

**Answer.**

(i) False because sides of a right triangle may have any length, so $\tan A$ may have any value.
(ii) True as $\sec A$ is always greater than 1 .
(iii) False as $\cos A$ is the abbreviation of cosine $\mathrm{A}$.
(iv) False as $\cot A$ is not the product of 'cot' and $\mathrm{A}$. 'cot' is separated from $\mathrm{A}$ has no meaning.
(v) False as $\sin \theta$ cannot be $> 1$.

# NCERT Class 10 Maths: Chapter 8 - Introduction to Trigonometry Solutions

**Ex 8.2 Question 1.**

Evaluate:

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

(iii) $\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

**Answer.**

(i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

(ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4} = 2$

(iii) $\frac{\cos 45^{\circ}}{\sec 30^{\circ}+\operatorname{cosec} 30^{\circ}}$

$=\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2}=\frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$

$=\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2+2\sqrt{3}}=\frac{\sqrt{3}}{\sqrt{2}\times 2(\sqrt{3}+1)}$

$=\frac{\sqrt{3}}{\sqrt{2}\times 2(\sqrt{3}+1)}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$

$=\frac{\sqrt{3}(\sqrt{3}-1)}{\sqrt{2}\times 2(3-1)}$

$=\frac{\sqrt{3}(\sqrt{3}-1)}{4\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$

$=\frac{3\sqrt{2}-\sqrt{6}}{8}$

(iv) $\frac{\sin 30^{\circ}+\tan 45^{\circ}-\cos ec 60^{\circ}}{\sec 30^{\circ}+\cos 60^{\circ}+\cot 45^{\circ}}$

$=\frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}=\frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}=\frac{3\sqrt{3}-4}{3\sqrt{3}+4}$

$=\frac{3\sqrt{3}-4}{3\sqrt{3}+4}\times\frac{3\sqrt{3}-4}{3\sqrt{3}-4}$

$=\frac{27+16-24\sqrt{3}}{27-16}$ $\left[\text{SSince }(a+b)(a-b)=a^2-b^2\right]$

$=\frac{43-24\sqrt{3}}{11}$

(v) $\frac{5\cos^2 60^{\circ}+4\sec^2 30^{\circ}-\tan^2 45^{\circ}}{\sin^2 30^{\circ}+\cos^2 30^{\circ}}$

$\frac{5\left(\frac{1}{2}\right)^2+4\left(\frac{2}{\sqrt{3}}\right)^2-(1)^2}{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$=\frac{5\times\frac{1}{4}+4\times\frac{4}{3}-1}{\frac{1}{4}+\frac{3}{4}}=\frac{\frac{5}{4}+\frac{16}{3}-1}{\frac{4}{4}}$

$=\frac{15+64-12}{12}=\frac{67}{12}$

**Ex 8.2 Question 2.**

Choose the correct option and justify:

(i) $\frac{2\tan 30^{\circ}}{1+\tan^2 30^{\circ}}=$

(A) $\sin 60^{\circ}$

(B) $\cos 60^{\circ}$

(C) $\tan 60^{\circ}$

(D) $\sin 30^{\circ}$

(ii) $\frac{1-\tan^2 45^{\circ}}{1+\tan^2 45^{\circ}}=$

(A) $\tan 90^{\circ}$

(B) 1

(C) $\sin 45^{\circ}$

(D) 0

(iii) $\sin 2A=2\sin A$ is true when $\mathbf{A}=$

(A) $0^{\circ}$

(B) $30^{\circ}$

(C) $45^{\circ}$

(D) $60^{\circ}$

(iv) $\frac{2\tan 30^{\circ}}{1-\tan^2 30^{\circ}}=$

(A) $\cos 60^{\circ}$

(B) $\sin 60^{\circ}$

(C) $\tan 60^{\circ}$

(D) None of these

**Answer.**

(i) (A) $\frac{2\tan 30^{\circ}}{1+\tan^2 30^{\circ}}$

$=\frac{2\times 1/\sqrt{3}}{1+(1/\sqrt{3})^2}$

$= \frac{2}{\sqrt{3}} \times \frac{3}{3+1} = \frac{\sqrt{3}}{2} = \sin 60^\circ$

(ii) (D) $\frac{1-\tan^2 45^\circ}{1+\tan^2 45^\circ} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

(iii) (A) Since $A = 0$, then

$\sin 2A = \sin 0^\circ = 0$ and $2 \sin A = 2 \sin 0^\circ$

$= 2 \times 0 = 0$

$\therefore \sin 2A = \sin A$ when $\mathrm{A} = 0$

(iv) (C) $\frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{3-1} = \sqrt{3} = \tan 60^\circ$$

**Ex 8.2 Question 3.**

If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = \frac{1}{\sqrt{3}}; 0^\circ B$, find $A$ and $B$.

**Answer.**

$\tan(A+B) = \sqrt{3}$

$\Rightarrow \tan(A+B) = \tan 60^\circ$

$\Rightarrow \mathrm{A} + \mathrm{B} = 60^\circ \ldots\ldots\ldots$ (i)

Also, $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan(A-B) = \tan 30^\circ$

$\Rightarrow \mathrm{A} - \mathrm{B} = 30^\circ$.

On adding eq. (i) and (ii), we get,

$2\ \mathrm{A} = 90^\circ \Rightarrow \mathrm{A} = 45^\circ$

On Subtracting eq. (i) and eq. (ii), we get

$2\ \mathrm{B} = 30^\circ \Rightarrow \mathrm{B} = 15^\circ$

**Ex 8.2 Question 4.**

State whether the following are true or false. Justify your answer.

(i) $\sin(A+B) = \sin A + \sin B$

(ii) The value of $\sin \theta$ increases as $\theta$ increases.

(iii) The value of $\cos \theta$ increases as $\theta$ increases.

(iv) $\sin \theta = \cos \theta$ for all values of $\theta$.

(v) $\cot A$ is not defined for $A = 0^\circ$.

**Answer.**

(i) False, because, let $\mathrm{A} = 60^\circ$ and $\mathrm{B} = 30^\circ$

Then, $\sin(A+B) = \sin(60^\circ + 30^\circ) = \sin 90^\circ = 1$

And $\sin A + \sin B = \sin 60^\circ + \sin 30^\circ = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

$\therefore \sin(A+B) \neq \sin A + \sin B$

(ii) True, because it is clear from the table below:

| $\theta$ | $0^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $90^\circ$ |
|---|---|---|---|---|---|
| $\sin \theta$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |

Therefore, it is clear, the value of $\sin \theta$ increases as $\theta$ increases.

(iii) False, because

| $\theta$ | $0^\circ$ | $30^\circ$ | $45^\circ$ | $60^\circ$ | $90^\circ$ |
|---|---|---|---|---|---|
| $\cos \theta$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |

It is clear, the value of $\cos \theta$ decreases as $\theta$ increases

(iv) False as it is only true for $\theta = 45^\circ$.

$\Rightarrow \sin 45^\circ = \frac{1}{\sqrt{2}} = \cos 45^\circ$

(v) True, because $\tan 0^\circ = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$

$= \frac{1}{0}$ i.e. undefined.

# NCERT Class 10 Maths: Chapter 8 - Introduction to Trigonometry Solutions

**Ex 8.3 Question 1.**

Express the trigonometric ratios $\sin A, \sec A$ and $\tan A$ in terms of $\cot A$

**Answer.**

For $\sin A$,
By using identity $\operatorname{cosec}^2 A - \cot^2 A = 1$

$$\Rightarrow \operatorname{cosec} A = 1 + \cot^2 A$$
$$\Rightarrow \frac{1}{\sin^2 A} = 1 + \cot^2 A$$
$$\Rightarrow \sin^2 A = \frac{1}{1 + \cot^2 A}$$
$$\Rightarrow \sin A = \frac{1}{\sqrt{1 + \cot^2 A}}$$

For $\sec A$,
By using identity $\sec^2 A - \tan^2 A = 1$

$$\Rightarrow \sec^2 A = 1 + \tan^2 A$$
$$\Rightarrow \sec^2 A = 1 + \frac{1}{\cot^2 A} = \frac{\cot^2 A + 1}{\cot^2 A}$$
$$\Rightarrow \sec^2 A = \frac{1 + \cot^2 A}{\cot^2 A}$$
$$\Rightarrow \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

For $\tan A$,
$\tan A = \frac{1}{\cot A}$

**Ex 8.3 Question 2.**

Write the other trigonometric ratios of A in terms of $\sec A$

**Answer.**

For $\sin A =$
By using identity, $\sin^2 A + \cos^2 A = 1$

$$\Rightarrow \sin^2 A = 1 - \cos^2 A$$
$$\Rightarrow \sin^2 A = 1 - \frac{1}{\sec^2 A} = \frac{\sec^2 A - 1}{\sec^2 A}$$
$$\Rightarrow \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

For $\cos A$,
$\cos A = \frac{1}{\sec A}$

For $\tan A$,
By using identity $\sec^2 A - \tan^2 A = 1$
$\Rightarrow \tan^2 A = \sec^2 A - 1$
$\Rightarrow \tan A = \sqrt{\sec^2 A - 1}$

For $\operatorname{cosec} A$,

$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$

$\Rightarrow \operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$

For $\cot A$,
$$\cot A = \frac{1}{\tan A}$$
$$\Rightarrow \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$
**Ex 8.3 Question 3.**

Evaluate:
(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$
(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

**Answer.**

(i) $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$
$$= \frac{\sin^2 63^\circ + \sin^2(90^\circ - 63^\circ)}{\cos^2(90^\circ - 73^\circ) + \cos^2 73^\circ}$$
$$= \frac{\sin^2 63^\circ + \cos^2 63^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$
$[\because \sin(90^\circ - \theta) = \cos\theta, \cos(90^\circ - \theta) = \sin\theta]$
$= \frac{1}{1} = 1 \left[\because \sin^2\theta + \cos^2\theta = 1\right]$
(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin 25^\circ \cdot \cos(90^\circ - 25^\circ) + \cos 25^\circ \cdot \sin(90^\circ - 25^\circ)$
$= \sin 25^\circ \cdot \sin 25^\circ + \cos 25^\circ \cdot \cos 25^\circ$

$[\because \sin(90^\circ - \theta) = \cos\theta, \cos(90^\circ - \theta) = \sin\theta]$
$= \sin^2 25^\circ + \cos^2 25^\circ = 1$
$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$
**Ex 8.3 Question 4.**

Choose the correct option. Justify your choice:
(i) $9\sec^2 A - 9\tan^2 A =$
(A) 1
(B) 9
(C) 8
(D) 0
(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta) =$
(A) 0
(B) 1
(C) 2
(D) none of these
(iii) $(\sec A + \tan A)(1 - \sin A) =$
(A) $\sec A$
(B) $\sin A$
(C) $\operatorname{cosec} A$

(D) $\cos A$

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A} =$
(A) $\sec^2 A$
(B) -1
(C) $\cot^2 A$

(D) none of these

Answer. (i) (B) $9 \sec^2 A - 9 \tan^2 A$

$= 9\left(\sec^2 A - \tan^2 A\right)$

$= 9 \times 1 = 9 \quad \left[\text{Since } \sec^2 \theta - \tan^2 \theta = 1\right]$

(ii) ( C) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$

$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$

$= \frac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta \cdot \sin\theta}$

$= \frac{\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta - 1}{\cos\theta \cdot \sin\theta}$

$= \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta \cdot \sin\theta}$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \frac{2\cos\theta\sin\theta}{\cos\theta \cdot \sin\theta} = 2$

(iii)(D) $(\sec A + \tan A)(1 - \sin A)$

$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$

$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$

$= \frac{1 - \sin^2 A}{\cos A} \quad \left[\text{Since } (a+b)(a-b) = a^2 - b^2\right]$

$= \frac{\cos^2 A}{\cos A}$

$= \cos A \left[\because 1 - \sin^2 A = \cos^2 A\right]$

(iv)(D) $\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A - \tan^2 A + \tan^2 A}{\operatorname{cosec}^2 A - \cot^2 A + \cot^2 A}$

$= \frac{\sec^2 A}{\operatorname{cosec} A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$

$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$

**Ex 8.3 Question 5.**

Prove the following identities, where the angles involved are acute angles for which the expressions are defined:

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

(v) $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$, using the identity $\operatorname{cosec} e^2 A = 1 + \cot^2 A$

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= 7 + \tan^2 A + \cot^2 A$

(ix) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \frac{1}{\tan A + \cot A}$

(x) $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

**Answer.**

(i) L.H.S. $(\operatorname{cosec}\theta - \cot\theta)^2$

$= \operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta\cot\theta \quad \left[\text{Since } (a-b)^2 = a^2 + b^2 - 2ab\right]$

$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - 2 \times \frac{1}{\sin\theta} \cdot \frac{\cos\theta}{\sin\theta}$

$= \frac{1+\cos^2\theta}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta}$

$= \frac{1+\cos^2\theta-2\cos\theta}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta} [\because a^2+b^2-2ab=(a-b)^2]$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta} =$ R.H.S.

(ii) L.H.S. $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2\theta+1+\sin^2\theta+2\sin A}{(1+\sin A)\cos A}$

$= \frac{\cos^2\theta+\sin^2\theta+1+2\sin A}{(1+\sin A)\cos A}$

$= \frac{1+1+2\sin A}{(1+\sin A)\cos A} [\because \sin^2\theta+\cos^2\theta=1]$

$= \frac{2+2\sin A}{(1+\sin A)\cos A} = \frac{2(1+\sin A)}{(1+\sin A)\cos A}$

$= \frac{2}{\cos A} = 2\sec A =$ R.H.S

(iii) L.H.S. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$

$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$

$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta-\sin\theta}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$

$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{(\sin\theta-\cos\theta)\left(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta\right)}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$[\because a^3-b^3=(a-b)\left(a^2+b^2+ab\right)]$

$= \frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta} [\because \sin^2\theta+\cos^2\theta=1]$

$= \frac{1}{\sin\theta\cos\theta} + \frac{\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} + 1 = 1 + \frac{1}{\sin\theta\cos\theta}$

$= 1 + \sec\theta \operatorname{cosec}\theta$

(iv) L.H.S. $\frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$

$= \frac{\cos A+1}{\cos A} \times \frac{\cos A}{1} = 1+\cos A$

$= 1+\cos A \times \frac{1-\cos A}{1-\cos A}$

$= \frac{1-\cos^2 A}{1-\cos^2 A}$ [Since $(a+b)(a-b)=a^2-b^2$]

$= \frac{\sin^2 A}{1-\cos A} =$ R.H.S.

(v) L.H.S. $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}$

Dividing all terms by $\sin A$,

$$=\frac{\cot A-1+\operatorname{cosec} A}{\cot A+1-\operatorname{cosec} A}=\frac{\cot A+\operatorname{cosec} A-1}{\cot A-\operatorname{cosec} A+1}$$

$$=\frac{(\cot A+\operatorname{cosec} A)-\left(\operatorname{cosec}^2 A-\cot^2 A\right)}{(1+\cot A-\operatorname{cosec} A)} \quad \left[\text{Since } \operatorname{cosec}^2 \theta-\cot^2 \theta=1\right]$$

$$=\frac{(\cot A+\operatorname{cosec} A)+\left(\cot^2 A-\operatorname{cosec}^2 A\right)}{(1+\cot A-\cos ec A)}$$

$$=\frac{(\cot \mathrm{A}+\operatorname{cosec} \mathrm{A})+(\cot \mathrm{A}+\operatorname{cosec} \mathrm{A})(\cot \mathrm{A}-\operatorname{cosec} \mathrm{A})}{(1+\cot \mathrm{A}-\operatorname{cosec} \mathrm{A})}$$

$=\cot A+\operatorname{cosec} A=$ R.H.S.

(vi) L.H.S. $\sqrt{\frac{1+\sin A}{1-\sin A}}$

$$=\sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}}$$

$$=\sqrt{\frac{(1+\sin A)^2}{1-\sin ^2 A}}\left[\because(a+b)(a-b)=a^2-b^2\right]$$

$$=\sqrt{\frac{(1+\sin A)^2}{\cos ^2 A}}\left[\because 1-\sin ^2 \theta=\cos ^2 \theta\right]$$

$$=\frac{1+\sin A}{\cos A}=\frac{1}{\cos A}+\frac{\sin A}{\cos A}$$

$=\sec A+\tan A=$ R.H.S.

(vii) L.H.S. $\frac{\sin \theta-2 \sin ^3 \theta}{2 \cos ^3 \theta-\cos \theta}$

$$=\frac{\sin \theta\left(1-2 \sin ^2 \theta\right)}{\cos \theta\left(2 \cos ^2 \theta-1\right)}$$

$$=\frac{\sin \theta\left(1-2 \sin ^2 \theta\right)}{\cos \theta\left[2\left(1-\sin ^2 \theta\right)-1\right]}$$

$\left[\because 1-\sin ^2 \theta=\cos ^2 \theta\right]$

$$=\frac{\sin \theta\left(1-2 \sin ^2 \theta\right)}{\cos \theta\left(2-2 \sin ^2 \theta-1\right)}$$

$$=\frac{\sin \theta\left(1-2 \sin ^2 \theta\right)}{\cos \theta\left(1-2 \sin ^2 \theta\right)}=\frac{\sin \theta}{\cos \theta}$$

$=\tan \theta=$ R.H.S

(viii) L.H.S. $(\sin A+\operatorname{cosec} A)^2+(\cos A+\sec A)^2$

$$=\left(\sin A+\frac{1}{\sin A}\right)^2+\left(\cos A+\frac{1}{\cos A}\right)^2$$

$$=\sin ^2 A+\frac{1}{\sin ^2 A}+2 \cdot \sin A \cdot \frac{1}{\sin A}+\cos ^2 A+\frac{1}{\cos ^2 A}+2 \cdot \cos A \cdot \frac{1}{\cos A}$$

$$=2+2+\sin ^2 A+\cos ^2 A+\frac{1}{\sin ^2 A}+\frac{1}{\cos ^2 A}$$

$$=4+1+\frac{1}{\sin ^2 A}+\frac{1}{\cos ^2 A}$$

$=5+\operatorname{cosec}^2 A+\sec ^2 A$

$=5+1+\cot ^2 A+1+\tan ^2 A$

$\left[\because \operatorname{cosec}^2 \theta=1+\cot ^2 \theta, \sec ^2 \theta=1+\tan ^2 \theta\right]$

$=7+\tan ^2 A+\cot ^2 A$

$=$ R.H.S.

(ix) L.H.S. $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)$

$$=\left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)$$

$$=\left(\frac{1-\sin ^2 A}{\sin A}\right)\left(\frac{1-\cos ^2 A}{\cos A}\right)$$

$$=\frac{\cos ^2 A}{\sin A} \times \frac{\sin ^2 A}{\cos A}=\sin A \cdot \cos A$$

$$=\frac{\sin A \cdot \cos A}{\sin ^2 A+\cos ^2 A}\left[\because \sin ^2 \theta+\cos ^2 \theta=1\right]$$

Dividing all the terms by $\sin A \cdot \cos A$,

$$=\frac{\frac{\sin A \cdot \cos A}{\sin A \cdot \cos A}}{\frac{\sin ^2 A}{\sin A \cdot \cos A}+\frac{\cos ^2 A}{\sin A \cdot \cos A}}$$

$$=\frac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}}$$

$$=\frac{1}{\tan A+\cot A}=\text { R.H.S. }$$

(x) L.H.S. $\left(\frac{1+\tan ^2 A}{1+\cot ^2 A}\right)=\frac{\sec ^2 A}{\operatorname{cosec} e^2 A}$

$\left[\because 1+\tan ^2 \theta=\sec ^2 \theta, 1+\cot ^2 \theta=\operatorname{cosec}^2 \theta\right]$

$$=\frac{1}{\cos ^2 A} \times \frac{\sin ^2 A}{1}=\tan ^2 A=\text { R.H.S. }$$

Now, Middle side $=\left(\frac{1-\tan A}{1-\cot A}\right)^2=\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$$=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$$

$$=\left(\frac{1-\tan \mathrm{A}}{\frac{-(1-\tan \mathrm{A})}{\tan \mathrm{A}}}\right)^2=(-\tan A)^2$$

$=\tan ^2 A=$ R.H.S.